Generation of Ultrashort pulses in a Highly Nonlinear Negative Index Material

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Abstract: In this paper, we consider the electromagnetic pulse propagation in a highly nonlinear negative index material wherein the pulse proagation is governed by cubic-qunitic nonlinear effects. By adopting coupled amplitude phase technique, we delineate the generation of ultrashort pulse in terms of bright soliton. In addition, we also calculate the peak power and pulse width required for generating this ultrashort pulse.

Keywords— Bright soliton, metamaterials, negative index materials, ultrashort pulses, modified nonlinear Schrodinger equation.

1 Introduction

Metamaterials (MMs) are artificially structured materials which exhibit electromagnetic properties, such as negative magnetic permeability, negative refraction, reversed Cerenkov radiation, sub-diffraction imaging, invisibility cloaking, that are not available in natural materials [1]. These unique properties led to several applications in science and technology [2]. For the first time, Veselago theoretically proposed that a material could also support the propgation of electromagnetic wave even if the material exhibits negative dielectric permitivty, ε, and negative magnetic permeability, μ. This kind of specail material possesses the negative refractive index and hence it is named as negative index material (NIM) [3]. One decade ago, these NIMs had first been practically realized in the microwave region using arrays of wire and split-ring resonators (SRRs) on the printed circuit board [4]. Various linear properties have been explored very well in the microwave regime. Recently, nonlinear effects have also been observed in NIMs when these materials are embedded with the weakly nonlinear dielectric [5]. First theoretical model was initiated by Scalora et al. who arrived at the generalized nonlinear Schrodinger equation which supports a wide class of solitary waves in NIMs [6]. Lazarides et al. derived the coupled nonlinear Schrodinger equations and theroretically demonstrated the formation of bright and dark solitons through Manakov model [7]. Wen et al. proposed a theoretical for generating ultrahort pulses with few optical cycles based on the Drude dispersive model [8].

In this paper, we study the generation ultrashort pulse in terms of bright soliton in nonlinear NIMs by using the modified nonlinear Schrodienger equation.

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2 Theoretical Model

The governing model for propagation of ultrashort pulses in nonlinear NIMs with Kerr and non-Kerr polarization (P_{NL}) is given as [9]

$$\frac{\partial E}{\partial \xi} + i\delta_1 \frac{\partial^2 E}{\partial \tau^2} - \delta_2 \frac{\partial^3 E}{\partial \tau^3} - i\sigma_0 |E|^2 E + \sigma_0 s_1 \frac{\partial(|E|^2 E)}{\partial \tau} - i\eta_0 |E|^4 E + \eta_0 s_3 \frac{\partial(|E|^4 E)}{\partial \tau} = 0, \quad (1)$$

Here δ_1 is the group velocity dispersion, δ_2 is the third order dispersion, σ_0 is the Kerr nonlinearity, s_1 is the self-steepening coefficient, η_0 is the quintinc nonlinearity, s_3 self-steepening parameter due to quintic nonlinearity. Based on equation (1), the modulation instability has been studied in nonlinear fiber Bragg gratings [10].

In order to generate the soliton type ultrashort pulses in NIMs, we consider the solution of the form of equation (1) as,

$$E(\xi, \tau) = U(\chi) \exp[i(k\xi - \Omega\tau)], \quad (2)$$

where $U(\chi)$, with $\chi = \tau - \Lambda\xi$, is a real quantity, Λ is a real parameter. k and Ω being the wavenumber and circular frequency, respectively. Substituting equation (2) in equation (1), we obtain,

$$\begin{aligned}&[-i\Lambda U_\chi + ikU] + i\delta_1[-\Omega^2 U - 2i\Omega U_\chi + U_{\chi\chi}] \\ &- \delta_2[i\Omega^3 U - 3\Omega^2 U_\chi - 3i\Omega U_{\chi\chi} + U_{\chi\chi\chi}] - i\sigma_0 U^3 \\ &+ \sigma_0 s_1[-i\Omega U^3 + 3U^2 U_\chi] - i\eta_0 U^5 \\ &+ \eta_0 s_3[-i\Omega U^5 + 5U^4 U_\chi] = 0,\end{aligned}$$

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Separating the real and imaginary parts, we have

$$\delta_2 U_{\chi\chi\chi} + (\Lambda - 2\Omega\delta_1 - 3\Omega^2\delta_2)U_\chi - 3\sigma_0 s_1 U^2 U_\chi - 5\eta_0 s_3 U^4 U_\chi = 0, \quad (3)$$

$$(\delta_1 + 3\delta_2\Omega)U_{\chi\chi} + (k - \delta_1\Omega^2 - \delta_2\Omega^3)U - \sigma_0(1 + s_1\Omega)U^3 - \eta_0(1 + s_3\Omega)U^5 = 0. \quad (4)$$

Where U_χ is the first order derivative with respect to χ and so on.

On integrating equation (3), we get

$$U_{\chi\chi} = -\frac{(\Lambda - 2\Omega\delta_1 - 3\Omega^2\delta_2)}{\delta_2}U + \frac{\sigma_0 s_1}{\delta_2}U^3 + \frac{\eta_0 s_3}{\delta_2}U^5 + c, \quad (5)$$

c is the integration constant, for simplicity c=0,

Rewriting equation (4) as,

$$U_{\chi\chi} = -\frac{(K - \delta_1\Omega^2 - \delta_2\Omega^3)}{(\delta_1 + 3\delta_2\Omega)}U + \frac{\sigma_0(1 + s_1\Omega)}{(\delta_1 + 3\delta_2\Omega)}U^3 + \frac{\eta_0(1 + s_3\Omega)}{(\delta_1 + 3\delta_2\Omega)}U^5. \quad (6)$$

Equation (5) and equation (6) are equivalent only under the following conditions:

$$\alpha = -\frac{(\Lambda - 2\delta_1\Omega - 3\delta_2\Omega^2)}{\delta_2} = -\frac{(K - \delta_1\Omega^2 - \delta_2\Omega^3)}{\delta_1 + 3\delta_2\Omega},$$

$$\beta = -\frac{\sigma_0(1 + s_1\Omega)}{\delta_1 + 3\delta_2\Omega} = -\frac{\sigma_0 s_1}{\delta_2},$$

$$\gamma = -\frac{\eta_0(1 + s_3\Omega)}{\delta_1 + 3\delta_2\Omega} = -\frac{\eta_0 s_3}{\delta_2}.$$

From the above relations, we find k and Ω as

$$k = \frac{(\Lambda - 2\delta_1\Omega - 3\delta_2\Omega^2)(\delta_1 + 3\delta_2\Omega)}{\delta_2} + \delta_1\Omega^2 + \delta_2\Omega^3, \quad (7)$$

$$\Omega = \frac{\delta_2 - s_1\delta_1}{2\delta_2 s_1}. \quad (8)$$

Equation (5) can be written as,

$$U_{\chi\chi} - \alpha U + \beta U^3 + \gamma U^5 = 0.$$

Integrating above equation, we get first order differential equation,

$$\left(\frac{dU}{d\chi}\right)^2 = 4\alpha U^2 - 2\beta U - \frac{2}{3}\gamma + c_1, \quad (9)$$

and c_1 is an integration constant.

Equation (9) is a first-order ordinary differential equation, we end up with the bright soliton pulses when the integration constant, $c_1=0$ and the same is given by,

$$U(\chi) = \sqrt{\frac{2\alpha}{\frac{\beta}{2} + \sqrt{\frac{\beta^2}{4} + \frac{4}{3}\alpha\gamma}\cosh[2\sqrt{\alpha}\chi]}}. \quad (10)$$

Now, by plugging this amplitude value in equation (2), we arrive at the generation of bright soliton type ultrashort pulse and the field envelope for the same is given by,

$$E(\xi,\tau) = \sqrt{\frac{2\alpha}{\frac{\beta}{2} + \sqrt{\frac{\beta^2}{4} + \frac{4}{3}\alpha\gamma}\cosh[2\sqrt{\alpha}(\tau - \Lambda\xi)]}} \times \exp[i(K\xi - \Omega\tau)]\ . \quad (11)$$

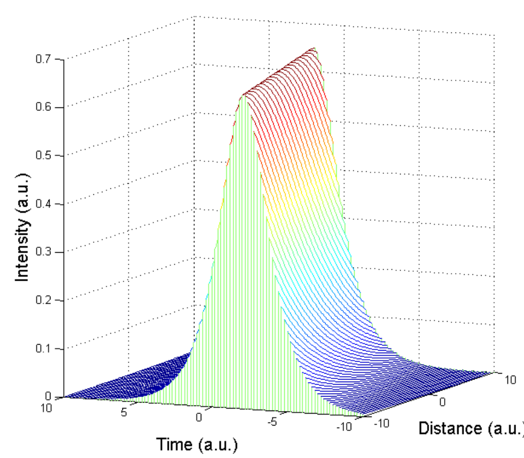

Fig. 1 Evolution of bright soliton type ultrashort pulse generated in a non-linear NIMs under the influence of higher order nonlinear effect.

Figure (1) portrays the three dimensional view of ultrashort pulse in terms of bright soliton pulse profile in the nonlinear NIMs. From the bright soliton pulse profile, we also calculate the important and interesting physical parameters such as soliton power and pulse width and they are found to be :

$$P_0 = \frac{2(2\delta_1\Omega + 3\delta_2\Omega^2 - \Lambda)}{\frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \frac{4}{3}(2\delta_1\Omega + 3\delta_2\Omega^2 - \Lambda)\gamma}}, \qquad (12)$$

$$T_0 = \frac{1}{\sqrt{2\delta_1\Omega + 3\delta_2\Omega^2 - \Lambda}}. \qquad (13)$$

with the known values for δ_1, δ_2 and s_1, we can calculate Ω using equation (8). Hence, for a given T_0, we can easily calculate the power required for generating the bright soliton type ultrashort pulse.

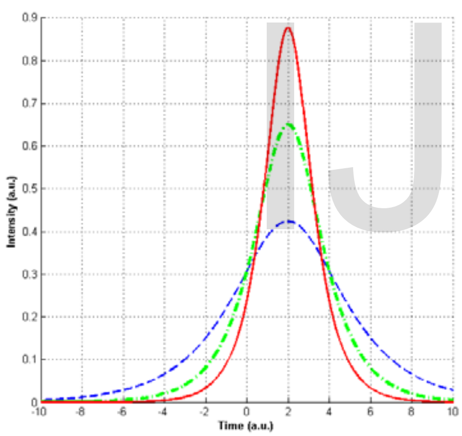

Fig. 2 Bright soliton for various values of input power: P_0 = 0.44 (blue curve), P_0 = 0.67 (green curve) and P_0 = 0.95 (red curve).

Figure (2) depicts the comparison of bright soliton pulse profiles for various peak powers when Λ = 0.2 and ξ = 10. It is obvious that the pulse width gets reduced as and when the power is increased.

3 Conclusion

We have successfully generated the bright soliton type ultrashort pulses in nonlinear negative index materials by solving the higher order nonlinear Schrodinger equation using coupled-amplitude phase method under the influence of cubic and quintic nonlinear effects. Besides, we have also calculated the minimum power required for realizing the ultrashort pulses. We do believe that the results presented in this paper would highly be useful for realizing optical switches and terahertz optical modulators.

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References

[1] J. B. Pendry, “Negative Refraction Makes a Perfect Lens”, Phys. Rev. Lett. 85, 3966(2000).

[2] Martin McCall, “Transformation optics and cloaking”, Contemporary Physics, 54, 273-286 (2013).

[3] V. G. Veselago, “The electrodynamics of substances with simultaneously negative values of ε and μ”, Sov. Phys. Usp. 10, 509(1968).

[4] D. R. Smith, W. Padilla, D.C. Vier, S. C. Nemat-Nasser and S. Schultz, “Composite Medium with Simultaneously Negative Permeability and Permittivity”, Phys. Rev. Lett. 84, 4184-4187 (2000).

[5] V. M. Shalaev, Nature Photon. 1 (2007) 41.

[6] M. Scalora, M. S. Syrchin, N. Akobek, E. Y. Poliakov, G. D’Aguanno, N. Mattiucci, M. J. Bloemer and A. M. Zheltikov, “Generalized Nonlinear Schrodinger Equation for Dispersive Susceptibility and Permeability: Application to Negative Inded Materials”, Phys. Rev. Lett. 95, 013902 (2005).

[7] N. Lazarides and G. P. Tsironis, “Coupled nonlinear Schrodinger field equations for electromagnetic wave propagation in nonlinear left-handed materials”, Phys. Rev. E 71, 036614 (2005).

[8] S. Wen, Y. Xiang, W. Su, Y. Hu, X. Fu and D. Fan, “Role of the anomalous self-steepening in modulation instability in negative-index material”, Opt. Express 14,1568-1575 (2006).

[9] H. Zhou, S. Wen, X. Dai, Y. Hu, Z. Tang, “Influence of nonlinear dispersion on modulation instability of coherent and partially coherent Ultrashort pulses in metamaterials”, Appl. Phys. B 87, 635-641 (2007).

[10] K. Senthilnathan, K. Porsezian and S. Devipriya, “Modulation Instability in Fiber Bragg Grating with Non-Kerr Nonlinearity”, IEEE J. Quantum Elect. 41, 789-796 (2005).